

# ANALYSIS OF MOVEMENT OF A SYSTEM WITH ELASTIC PENDULUM

ANALYSIS OF THE INFLUENCE OF PENDULUM AMPLITUDE  
ON MOVEMENT OF A PENDULUM SYSTEM  
DURING SLIDING FRICTION AND ROLLING FRICTION

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## ABSTRACT

This paper analyzes the sliding friction and the rolling friction of a moving system, driven by oscillations of the pendulum with an elastic pendulum handle, which was mounted on the system. The aim was to observe differences in the systems under the effect of impact forces (from plastic and steel calipers) on the metal ball, as well as to analyze and compare the results obtained.

*Keywords: elastic pendulum, oscillations, amplitude, movement, sliding friction, rolling friction.*

## INTRODUCTION

A mathematical pendulum is an idealization, working on the assumption that:

- The rod or cord on which the bob is hanged *doesn't have a mass, is inextensible and always remains taut*. The mass of the pendulum bob is concentrated in a point.
- Motion in 2-D, i.e. the bob does not trace an ellipse
- The motion *does not lose energy to friction*
- Air resistance is *nonexistent* <sup>[1]</sup>

A mathematical pendulum is defined as:

“A body suspended from a fixed point so that it can swing back and forth under the influence of the gravity. A simple pendulum consists of a bob (weight) suspended at the end of a string. The periodic motion of a pendulum is constant, but can be made longer or shorter by increasing or decreasing the length of the string. A change in the mass of the bob alone does not affect the period.” <sup>[2]</sup>

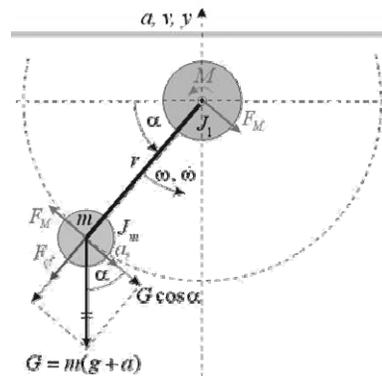


Figure 1. Mathematical pendulum

In modern mechanics, analysis of the pendulum movement takes assumptions that the movements are very small oscillations (*less than  $5^\circ$* ), while the movement of the pendulum at higher amplitudes is quite insufficiently explored, both theoretically and practically.

During the oscillation of the pendulum, kinetic and potential energy are transforming one into the other, and the forces change their magnitudes and directions. Because of the centrifugal force in the lower position, the resultant force acting on the pendulum culminates (*due to the maximum speed of the movement*), and the effect of the force of gravity is in its maximum, so that the pendulum moves down and then moves upward in the direction of vertical plane. <sup>[1]</sup>

Surplus of energy, due to these inertial forces and gravitational potential can be explained by the additional acceleration due to gravity potential. <sup>[3]</sup>

## EQUIPMENT AND MEASURING INSTRUMENTS USED IN EXPERIMENT

In this paper, an attempt has been made, to analyze the physical phenomena that resulted from the force applied on the pendulum, and to compare the results with a variation of the amplitude of oscillations from 0 to 121 mm.

Devices used in the experiment:

1. Elastic pendulum and a wooden block *(Figure 2)*
2. Measuring instruments - calipers (plastic and steel) *(Figure 3)*
3. Cart
4. Universal sliding gauge
5. Ruler



*Figure 2. Elastic pendulum system*



*Figure 3. Measuring instruments - calipers (steel and plastic)*

## THE COURSE OF EXPERIMENT AND THE RESULTS

This experiment is further analyzed in Veljko Milkovic's video

“Power of the pendulum - proof of ultra-efficiency?”

<http://www.youtube.com/watch?v=YnetjttZlRk>

In the process of measuring, first is defined distance " $D$ " which is the distance from the center of metal ball to the nearest side of the wooden block, to represent reference size of amplitude (*Figure 4*).

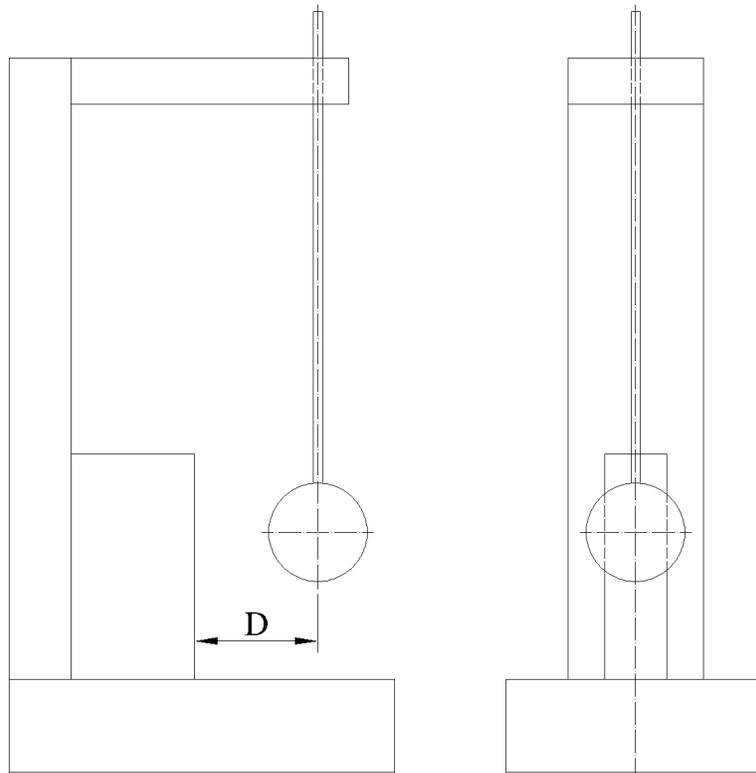


Figure 4. Model of Pendulum System Depicting Variable " $D$ "

The experiment was carried out in two main stages; In the first stage was analyzed sliding friction and in second stage was analyzed rolling friction. In both stages a caliper was used for hitting a metal ball. After hitting the ball, a measuring of the displacement of the entire system has been done.

Each measurement has been repeated three times and after that parameter " $D$ " was varied (*done by rotating the wooden block*). We have listed the mean value of the results in this paper.

The experiment was first performed on a stable surface (*analysis of sliding friction*), then on a mobile base (*a cart - rolling friction*), in order to compare the results and observe phenomena which accompany the oscillations of a pendulum.

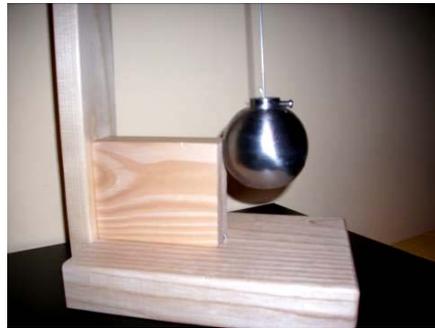
Video of how the experiment was conducted is available at:

“Expertise of Veljko Milkovic's elastic string pendulum experiment“

<http://www.youtube.com/watch?v=O1hvIoNn6I8>

## ***SLIDING FRICTION***

The following pictures are showing the cases of variation of parameter " $D$ ", for the analysis of different results (*Figures 5-8*).



*Figure 5. Elastic pendulum  $D = 32.2$  mm (blocked pendulum)*



*Figure 6. Elastic pendulum  $D = 43.2$  mm (minimum amplitude)*



*Figure 7. Elastic pendulum  $D = 85$  mm (mean amplitude)*

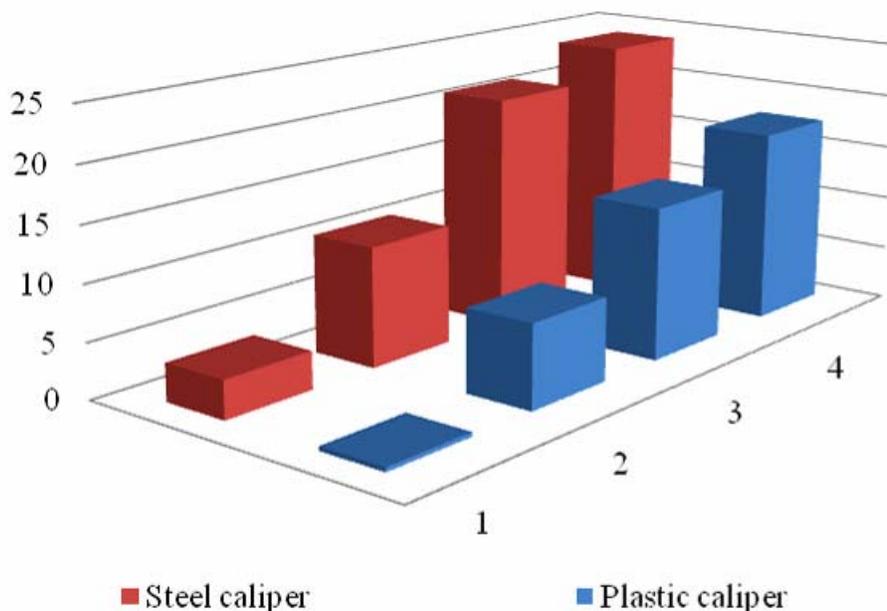


*Figure 8. Elastic pendulum  $D = 121$  mm (maximum amplitude)*

The results are presented in the table below (*Table 1 and Chart 1*) and, as shown, it is evident that the use of steel caliper causes greater movement of the pendulum system, which agrees with the assumption that steel caliper "stores" more energy than plastic one, because it has a greater step and it gives a greater resistance on the impact.

№	D [mm]	Steel caliper [mm]	Plastic caliper [mm]
1	32.2	3.46	0.40
2	43.2	10.87	7.53
3	85.0	21.21	13.77
4	121.0	23.88	17.53

*Table 1. Sliding friction (distance in mm)*



*Chart 1. Sliding friction (distance in mm)*

Input energy is spent on overcoming air resistance, friction of the pendulum system on the surface, plastic and elastic deformation of pillars of the pendulum, thermal energy is generated in the zones of impact and so on.

It is important to note that the remaining energy of the pendulum, after the termination of the force and the movement of the system (pendulum continues to perform oscillatory motion), is *about 80%*, which indicates the additional effect of the gravitational force on a metal ball.

## ***ROLLING FRICTION***

In the second part of an experiment, an elastic pendulum system has been placed on a cart with four wheels (*Figure 9*), so that the effect of a rolling friction can be simulated, observed and measured. The experiment has been conducted in a similar way as the previous one (varying “*D*” and using the same plastic and metal calipers with the same amount of “*accumulated*” energy).



*Figure 9. Pendulum system on cart*

It was expected that the movement of the system is greater than in the previous case, because it is known that the rolling friction is smaller than the sliding friction. The results met the expectations and forecasts.

Unlike the system in which sliding friction was analyzed it was noted that after the cessation of the force and the movement of the cart, the pendulum oscillated considerably shorter, which leads to the conclusion that in the system remained much less energy for the oscillations (*about 30%*), as a consequence of dampening oscillations caused by the movement of cart.

The measurement was also performed for four characteristic cases of the parameter " $D$ "



Figure 10. Elastic pendulum on a cart  $D = 32.2$  mm (blocked pendulum)



Figure 11. Elastic pendulum on a cart  $D = 43.2$  mm (minimum amplitude)



Figure 12. Elastic pendulum on a cart  $D = 85$  mm (mean amplitude)

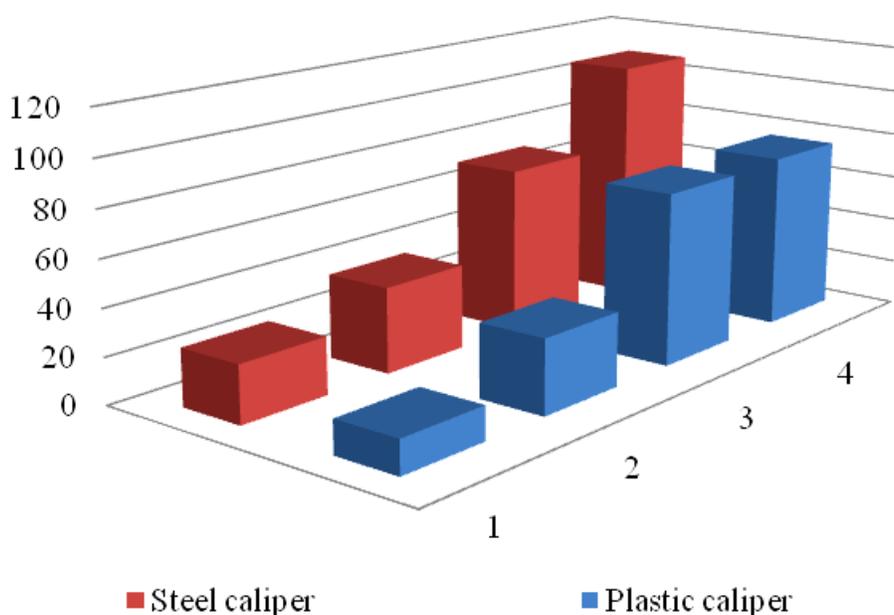


Figure 13. Elastic pendulum on a cart  $D = 121$  mm (maximum amplitude)

The measurement results are shown in *Table 2* and *Chart 2*.

№	$D$ [mm]	Steel caliper [mm]	Plastic caliper [mm]
1	32.2	24.8	14.5
2	43.2	37.0	31.9
3	85.0	72.0	74.2
4	121.0	107.3	75.5

*Table 2. Pendulum system on a cart (distance in mm)*



*Chart 2. Pendulum system on a cart (distance in mm)*

*Table 2* and *Chart 2* show that an increase of the amplitude of oscillations leads to increased distance passed by the cart, and when compared with the results for the system with sliding friction, it is evident that rolling friction ensures greater distances passed by the system.

## CONCLUSION AND ANALYSIS OF RESULTS OBTAINED

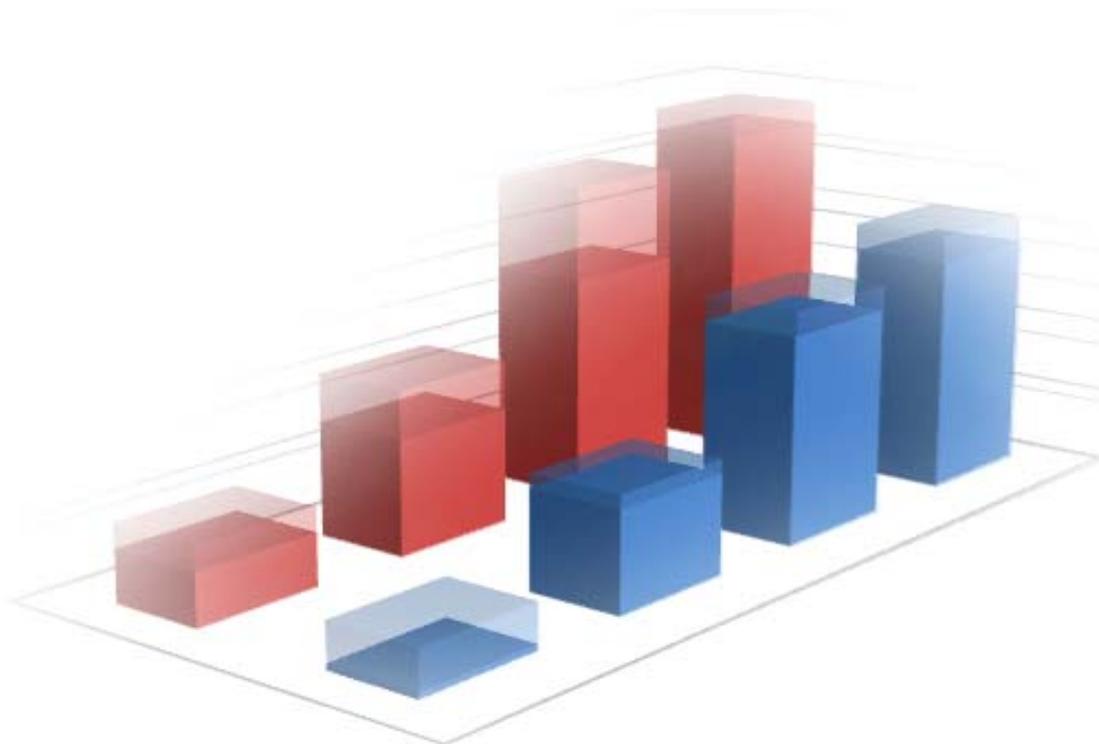
The experiment was performed with a goal to analyze events of large amplitude of oscillations of the pendulum, which modern mechanics and physics have not sufficiently explored, and that this should not be an obstacle for further investigations by persons interested in this field.

A very interesting phenomenon, concerning the distance passed, was noticed after varying distance " $D$ ", which brings us to the conclusion that *greater oscillations give greater movement and also generate a greater amount of energy* and that the gravitational force seems to "add" some energy to the system. This comment should be accepted with caution because the scientific community generally accepts the law of conservation of energy. But the experiment shows that it's evident that our gravitational force "*helps*" us to achieve greater movement of the pendulum system in both cases (*sliding and rolling*).

Like in any experiment, it is possible to improve a certain segments, so it was also noticed that in this experiment certain improvements can be made in the form of exclusion of the errors caused by the human factor, which is mostly related to the introduction of automated (*robotic*) energy transfer system to a pendulum, which would provide greater reproducibility of results and greater accuracy. It is also necessary to expand the analysis of large amplitude oscillations to find a critical point at what it is profitable to invest energy.

As in comments about academician Milkovic's work, there may be questions asked about static friction in the calipers or other issues, but that will probably be analyzed in the future papers on this matter. All comments are welcome as well as cooperation with all people of good will.

I hope that this will encourage at least small part of the world's population to realize the advantages of using gravitational energy as a clean energy source, and we will thus take a step together for a better and cleaner world.



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